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Traffic accidents in a cellular automaton model with a speed limit zone

Wei Zhang, Xian-qing Yang, Da-peng Sun, Kang Qiu and Hui Xia

College of Science, China University of Mining and Technology, Xuzhou 221008, People's Republic of China

E-mail: xianqyang@sina.com.cn

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Abstract
In this paper, we numerically study the probability P_{ac} of the occurrence of car accidents in the Nagel–Schreckenberg (NS) model with a speed limit zone. Numerical results show that the probability for car accidents to occur P_{ac} is determined by the maximum speed v'_{max} of the speed limit zone, but is independent of the length L_v of the speed limit zone in the deterministic NS model. However in the nondeterministic NS model, the probability of the occurrence of car accidents P_{ac} is determined not only by the maximum speed v'_{max}, but also the length L_v. The probability P_{ac} increases accordingly with the increase of the maximum speed of the speed limit zone, but decreases with the increase of the length of the speed limit zone, in the low-density region. However in the case of $v'_{max} = 1$, the probability P_{ac} increases with the increase of the length in the low-density region, but decreases in the interval between the low-density and high-density regions. The speed limit zone also causes an inhomogeneous distribution of car accidents over the whole road. Theoretical analyses give an agreement with numerical results in the nondeterministic NS model with $v'_{max} = 1$ and $v_{max} = 5$.

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1. Introduction

Recently, cellular automaton (CA) traffic flow problems have attracted much attention in a community of physicists because of the observed nonequilibrium phase transitions and various nonlinear dynamical phenomena. The advantages of the approaches show the flexibility to adapt complicated features observed in real traffic [1–3]. The Nagel–Schreckenberg (NS) model [4] and the Fukui–Ishibasi (FI) model [5] are two basic CA models describing one-lane traffic flow. Based on the two basic models, many CA models have been extended to investigate real traffic systems such as road blocks and hindrances, two-level crossing, highway junctions, etc [6].

Traffic jams and traffic accidents have become more and more significant problems in modern society. Presently, the CA models have been extended to investigate the traffic accidents [7–17]. Simulations of the probability for car accidents to occur have been offered in the NS models with periodic and open boundary [7–13], FI model [14], velocity effect CA model [15] and two-lane CA model [16], with the help of the conditions for the occurrence of car accidents proposed by Boccara *et al* [7]. Recently, the effects of a type of quenched randomness on car accidents have been investigated in the NS model [17].

In this paper, we study the probability for car accidents to occur in the NS model considering the speed limit zones. In the real traffic systems, speed limit signs are usually posted to warn drivers of potential dangerous situations, such as sharp curves, tunnels, bridges, etc, requiring a reduction in the maximum speed. Thus, in the speed limit zones, the speed of cars is reduced and the maximum velocity of cars differs from that in the normal zones, if the drivers respect the traffic rules. On the other hand, the speed limit zone is not only different from particlewise quenched disorder but sitewise randomness. For the particlewise disorder, a higher limit speed is assigned to a fraction of cars chosen randomly while a lower limit velocity is assigned to the other [18–20], and for sitewise disorder, vehicles move with different behaviours only at the defects [21–25]. But in the speed limit zone, vehicles can move with different maximum speed from the normal region. The differences may result in changes of the traffic flow and the probability for car accidents to occur. To our knowledge, the flow and the accident probability for the model with the speed limit zones have not been explored so far, and should be further investigated.

The paper is organized as follows. Section 2 is devoted to the description of the model and the conditions for the occurrence of car accidents. In section 3, the numerical studies of the car accidents are given, and the influences of the maximum speed and the length of the speed limit zone on the probability of car accidents are considered. The accident probability at sites are also presented. The results are summarized in section 4.

2. Model and car accidents

The model is defined on a single lane road consisting of L cells of equal size numbered by $i = 1, 2, \ldots, L$ and the time is discrete. Each site can be either empty or occupied by a car with the speed $v = 0, 1, 2, \ldots, v_{\max}$, where $v_{\max}$ is the speed limit in the normal zone. Let $x(i,t)$, $v(i,t)$ and $v'_{\max}$ denote the position, the velocity of the ith car at time t and the speed limit in the speed limit zone, respectively. The number of empty cells in front of the ith car is denoted by $d = x(i+1) - x(i) - 1$. The following four steps for all cars updated in parallel with periodic boundary.

(1) Acceleration:
IF (the car in the speed limit zone) THEN
$v(i, t+1/3) \to \min[v(i,t)+1, v'_{\max}]$
ELSE
$v(i, t+1/3) \to \min[v(i,t)+1, v_{\max}]$
END IF
(2) Slowing down:
$v(i, t+2/3) \to \min[v(i, t+1/3), d(i,t)]$
(3) Stochastic braking:
$v(i, t+1) \to \max[v(i, t+2/3) - 1, 0]$ with the probability p
(4) Movement: $x(i, t+1) \to x(i,t) + v(i, t+1)$

A region is designed as the speed limit zone, where each vehicle can only move with the maximum velocity $v'_{\max}$. Usually, the maximum velocity $v'_{\max}$ in the speed limit zone is smaller than the maximum speed $v_{\max}$ in the normal region. The length of the zone is denoted by L_v. Apparently, if $L_v = 0$, the present model returns to the basic NS model with the speed limit $v_{\max}$; if $L_v = 1$, the model is similar with the NS model with the defect, and when the speed limit zone is extended to the entire road, i.e., $L_v = L$, the above model will revert to the NS model with the maximum speed $v'_{\max}$. Therefore the NS model with the speed limit zone is an expansion of the basic NS model.

In the NS model, car accidents will not occur, because the second rule of the update is designed to avert accidents. In real traffic, however, car accidents occur most likely, due to the careless driving of the drivers who do not keep the safety distance. Boccara *et al* [7] proposed the following three conditions to determine whether car accidents occur: (i) $d \leqslant v_{\max}$, (ii) $v(i+1,t) > 0$ and (iii) $v(i+1,t+1) = 0$. Considering the stochastic braking , one of our authors has presented the modified conditions which are applicable for both deterministic and nondeterministic systems to correctly determine the probability for the car accidents to occur [14]. The first condition is that over the iterations of the rules 1–3, the velocity of the car is exactly equal to the number of empty cells in front of it, which means that the car can reach the position of the car ahead if the velocity of the car driven by the careless driver increases by one unit. The second condition is that the car ahead is moving. The third condition is that the moving car ahead is abruptly stopped. If the above three conditions are satisfied simultaneously and the velocity of following cars increase by one with the probability p', the collision between the two cars will happen.

In this paper, we utilize these conditions to investigate the probability of traffic accidents. In the process of simulations, car accidents do not really take place. When the three necessary conditions are met simultaneously, the dangerous situations of the occurrence of car accidents exist. The dangerous situations are calculated and considered as the signal of the occurrence of car accidents. Generally, the probability per car per time step for car accidents to occur is denoted by P_{ac}. The probability of car accidents can be scaled by the parameter p' because the occurrence of traffic accidents is proportional to the occurrence of dangerous situations, and the proportional constant is p'. The system size $L = 1000$ is selected, and the results are obtained by averaging over 30 initial configurations and 10^4 time step after discarding 10^4 initial transient states.

3. Numerical results

3.1. Effects of the speed limit in the speed limit zone

First, we investigate the influences of the speed limit in the limit zone on the probability P_{ac} for the occurrence of car accidents in the deterministic case. In the deterministic NS model with a speed limit region, the stochastic braking is not considered, i.e. $p = 0$. Figure 1 shows the accidents probability P_{ac} as a function of ρ for various values of L_v and $v'_{\max}$. As shown in figure 1, we find that the probability of the occurrence of the car accidents P_{ac} is only determined by the speed limit in the speed limit zone $v'_{\max}$, and is independent of the length of the speed limit region L_v. The accident probability P_{ac} is the same as the probability P_{ac} in the basic NS model with the maximum speed $v'_{\max}$ of the speed limit zone.

The above phenomenon can be understood. Fundamental diagram of the NS model with speed limit zone shows that the traffic flow is maximum and independent of the density of the vehicles over the interval $\rho_{c1} \leqslant \rho \leqslant \rho_{c2}$, due to the traffic flow limiting factor of the speed limit zone. The saturated flow in the fundamental diagram is the maximum flow corresponding

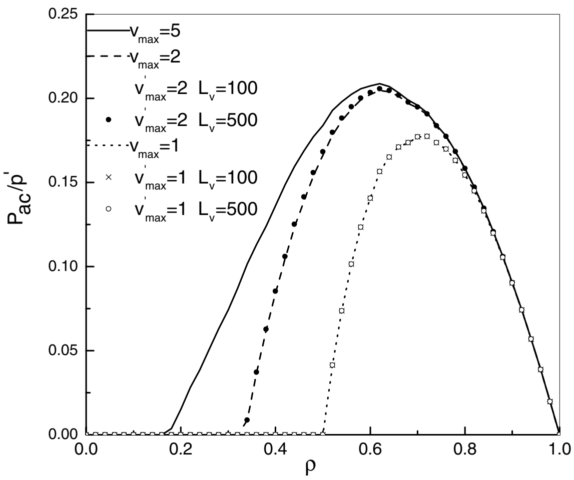

Figure 1. Probability P_{ac} (scaled by p') as a function of the density ρ in the deterministic NS model with a speed limited area with $v_{max} = 5$ for various values of the speed limit v'_{max} and the length L_v.

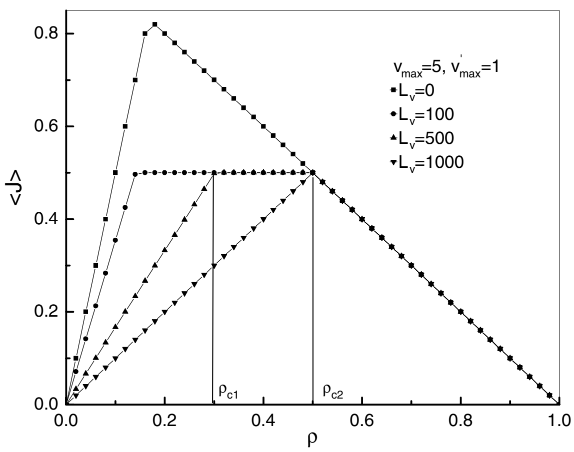

Figure 2. Fundamental diagram of the NS model with $v_{max} = 5$ and $v'_{max} = 1$ in the deterministic case.

to the NS model with the speed limit v'_{max}. When the density ρ is smaller than the critical density ρ_{c1}, the flow increases with the increase of the density; when $\rho > \rho_{c2}$, the traffic flow decreases as the density increases. Increasing the length of the speed limit zone only influences on the value of the critical density ρ_{c1}. Therefore in the density interval $0 < \rho < \rho_{c2}$, the traffic flow is free and no stopped cars exist. These results are shown in figure 2.

According to the previous studies [14], the probability is directly relative to the traffic flow and the stopped cars. When $\rho \leqslant \rho_{c2}$, no stopped cars occur, thus there are no car accidents. While $\rho > \rho_{c2}$, the stopped cars occur, hence cause the occurrence of the car accidents. The critical density ρ_{c2} is only determined by the maximum speed v'_{max}, and equals $1/(1 + v'_{max})$, as shown in figure 2.

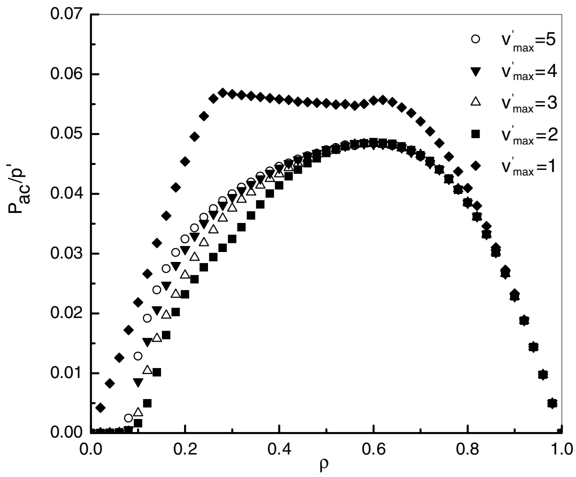

Figure 3. Probability P_{ac} (scaled by p') as a function of the density ρ in the nondeterministic NS model with the speed limit zone for the case of $p = 0.5$, $L_v = 500$ and $v_{\max} = 5$, for various values of the speed limit $v'_{\max}$.

The probability P_{ac} exhibits different effects of the speed limit $v'_{\max}$ when the stochastic braking behaviours of drivers are considered. Figure 3 shows the relations of the probability P_{ac} to the density ρ with various values of the speed limit $v'_{\max}$ in the case of $v_{\max} = 5$. The braking probability $p = 0.5$. As shown in figure 3, there is a critical density below which no car accidents occur because of no stopped cars. When the car density ρ is small, the probability P_{ac} decreases with the decrease of the limit speed $v'_{\max}$, owing to the traffic flow suppressed by the speed limit zone; while ρ is larger, the values of the probability for the occurrence of car accidents P_{ac} shrink to one curve. The exception is the case of $v'_{\max} = 1$. In figure 3, for the case of $v'_{\max} = 1$, no critical density exists. The accident probability P_{ac} shows the approximate plateau over a certain density region. In the low-density region, the accident probability P_{ac} increases with the increase of the density. But in the high-density region, the probability P_{ac} decreases with the increase of the density.

3.2. Effects of the length of the speed limit zone

The probability P_{ac} for car accidents is not only related to $v'_{\max}$, but also determined by the length L_v of the speed limit zone when the stochastic braking probability $p \neq 0$. Figure 4 exhibits the relation of P_{ac} to ρ with different value of the length of the speed limited area in the case of $v'_{\max} = 2$ and $v_{\max} = 5$. In figure 4, we observe that the critical density for the car accidents to occur shifts towards the right, and the probability of the occurrence of car accidents P_{ac} decreases in the case of low density of cars, when the length of speed limit zone increases. As the length of speed limit zone increases, the traffic system is gradually dominated by the NS model with speed limit $v'_{\max}$. According to the previous studies [8, 12], the critical density for the occurrence of car accidents decreases with the increase of the speed limit in the basic NS model, and the probability for car accidents to occur increases accordingly in the low density of cars. Therefore, with the increase of the length of the speed limit zone, the critical density of car accidents increases.

But, for the case of $v'_{\max} = 1$, there is no critical density because the stopped cars always exist only if $p \neq 0$. With the increase of the length of the speed limit zone, the probability

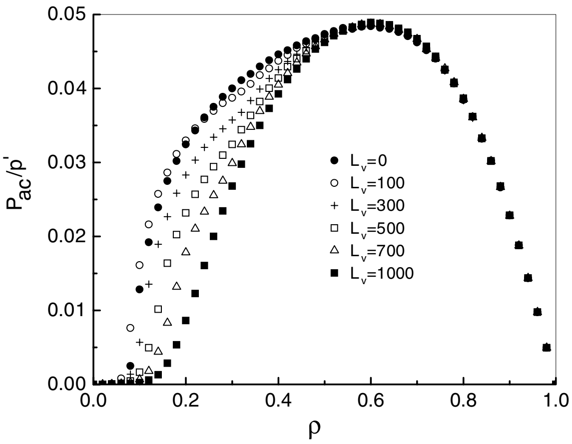

Figure 4. Probability P_{ac} (scaled by p') as a function of the density ρ in the nondeterministic NS model with the speed limit area for the case of $p = 0.5$, $v_{max} = 5$ and $v'_{max} = 2$, for various values of the length L_v.

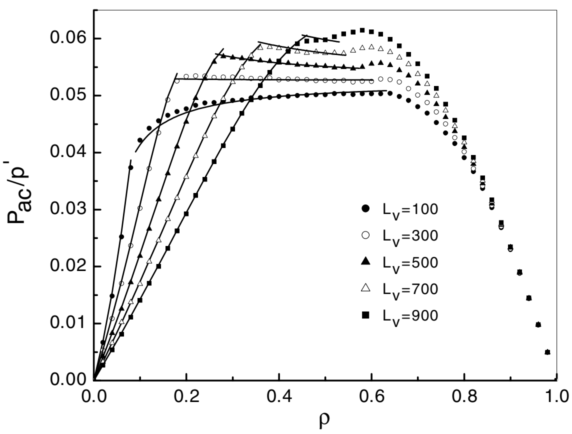

Figure 5. Probability P_{ac} (scaled by p') as a function of the density ρ in the nondeterministic NS model with the speed limit zone for the case of $p = 0.5$, $v_{max} = 5$ and $v'_{max} = 1$, for various values of the length L_v. Symbol data are obtained from computer simulations, and solid line corresponds to analytic results.

P_{ac} for car accidents to occur decreases in the low-density region; however, in the interval between the low and high densities, the probability P_{ac} increases. The results are shown in figure 5. The above results above can be quantitatively explained. When the stochastic braking is considered, the fundamental diagram of the present model with $v'_{max} = 1$ is similar with that for the deterministic case, these data are not shown. When the density is very low, i.e., $0 \leqslant \rho \leqslant \rho_{c1}$, the traffic system shows a free-moving phase, and traffic flow is given as

$$\langle J \rangle = \frac{1 - \sqrt{1 - 4q(1 - \rho_v)\rho_v}}{2} = (v_{max} - p)\rho_n, \tag{1}$$

where $\rho_n = N_n/L_n$, and $\rho_v = N_v/L_v$ denotes the density in the normal region and in the speed limit zone, respectively. In equation (1), $q = 1 - p$. Equation (1) is obtained by the relation that the traffic flows is same across the entire system. The second term is the flow in the speed limit zone, and the third is the flux in the normal region. Conservation of the cars demands that

$$L_v\rho_v + (L - L_v)\rho_n = \rho L. \tag{2}$$

Based on equations (1) and (2), we obtain the density of the speed limit zone:

$$\rho_v = B - \sqrt{B^2 - C[A^2 - 4(\rho - A/2)^2]}/(2C), \tag{3}$$

where $A = \frac{L-L_v}{(v_{\max}-p)L}$, $B = qA^2 - 2(\rho - A/2)\frac{L_v}{L}$ and $C = qA^2 - \left(\frac{L_v}{L}\right)^2$. When the density of the speed limit zone $\rho_v = 1/(v'_{\max} + 1) = 1/2$, corresponding to the maximum traffic flow in the speed limit zone, further increase of the car density results in the plateau of traffic flow in the fundamental diagram. Substituting the relation of $\rho_v = 1/2$ into equation (3), we derive the critical density ρ_{c1} as follows:

$$\rho_{c1} = \frac{L_v}{L}\left(\frac{1}{2} - \frac{1-\sqrt{p}}{2(v_{\max} - p)}\right) + \frac{1-\sqrt{p}}{2(v_{\max} - p)}. \tag{4}$$

According to the definition of the probability of the occurrence of car accidents, the probability P_{ac} is given as

$$\begin{aligned} P_{\mathrm{ac}} &= \frac{P_{\mathrm{ac}}^v N_v + P_{\mathrm{ac}}^n N_n}{N} \\ &= \frac{P_{\mathrm{ac}}^v \rho_v L_v + P_{\mathrm{ac}}^n \rho_n (L - L_v)}{\rho L} \\ &= \frac{L_v\rho_v}{L\rho} P_{\mathrm{ac}}^v + \left(1 - \frac{L_v}{L}\right)\frac{\rho_n}{\rho} P_{\mathrm{ac}}^n, \end{aligned} \tag{5}$$

where P_{ac}^v and P_{ac}^n denote the probability for car accidents to occur per time step per car in the speed limit zone and in the normal region, respectively. Equation (5) indicates that the number of car accidents in the entire system is equal to the number of car accidents in both the speed limit zone and the normal region. In the case of $v'_{\max} = 1$, the probability P_{ac}^v for the car accidents to occur in the speed limit region reads as [14]

$$P_{\mathrm{ac}}^v = p' \frac{[1 - \sqrt{1 - 4q(1-\rho_v)\rho_v}][2q\rho_v + \sqrt{1 - 4q(1-\rho_v)\rho_v} - 1]}{4\rho_v^2}. \tag{6}$$

When $\rho < \rho_{c1}$, no car accidents occur in the normal region because of no stopped cars, thus the probability $P_{\mathrm{ac}} = \frac{L_v\rho_v}{L\rho} P_{\mathrm{ac}}^v$. Substituting formula (3) into (6), we can obtain the relation of the probability P_{ac} to the car density ρ. Comparison of our prediction for the probability P_{ac} with the computer simulations shows excellent agreement (see figure 5).

When the density further increases over the critical density ρ_{c1}, traffic jams begin to emerge in the normal area. In the density interval $\rho_{c1} \leqslant \rho \leqslant \rho_{c2}$, the density of the speed limit zone $\rho_v = 1/2$, and the traffic flow remains in the saturated value, and is given below:

$$\langle J \rangle_s = \frac{1 - \sqrt{1 - 4q(1-\rho_v)\rho_v}}{2} = (1 - \sqrt{p})/2. \tag{7}$$

In the normal area, the traffic exhibits macroscopic phase segregation into high-density and low-density regions. In the low-density region, the traffic exhibits a free flow, but in the high-density region the traffic shows jamming phase. Therefore, the traffic flow reads as

$$\langle J \rangle_n = (v_{\max} - p)\rho_n^f = \frac{\rho_n^j}{T_w + 1}, \tag{8}$$

where ρ_n^f and ρ_n^j are the density in the free flow region and congested region of the normal area, respectively. Equation (8) indicates conservation of the flux in the normal region. The second term is the flux in the low-density region of the normal zone; the third is the flow in the high-density region of the normal zone [24]. T_w indicates the average waiting time T_w of the first vehicle at the boundary between the normal and speed limit regions. According to the definition of the average velocity of cars, the waiting time T_w is obtained as follows:

$$T_w = \frac{1}{\langle v\rangle_v} = \frac{2\rho_v}{1-\sqrt{1-4q(1-\rho_v)\rho_v}} = \frac{1}{1-\sqrt{p}}, \tag{9}$$

where the car density $\rho_v = 1/2$ corresponding to the maximum flow in the speed limit zone. Using equations (7)–(9), we obtain the car density in the low-density and high-density regions, respectively:

$$\rho_n^j = \frac{2-\sqrt{p}}{2}, \qquad \rho_n^f = \frac{1-\sqrt{p}}{2(v_{\max}-p)}. \tag{10}$$

And in the entire system, the conservation of the vehicles demands that

$$L_v\rho_v + \left(L - L_v - L_n^j\right)\rho_n^f + L_n^j\rho_n^j = \rho L. \tag{11}$$

Thus the length of the high-density region of the normal area is derived:

$$\begin{aligned} L_n^j &= \frac{\rho L - \rho_v L_v - (L-L_v)\rho_n^f}{\rho_n^j - \rho_n^f} \\ &= \frac{\rho L - L_v/2 - (L-L_v)\frac{1-\sqrt{p}}{2(v_{\max}-p)}}{\frac{2-\sqrt{p}}{2} - \frac{1-\sqrt{p}}{2(v_{\max}-p)}} \end{aligned} \tag{12}$$

Equation (12) indicates that the length L_n^j increases linearly with the increase of the density ρ in the density interval $\rho_{c1} \leqslant \rho \leqslant \rho_{c2}$. In the high-density region of the normal area, the density of stopped cars can be obtained by considering that the probability for a site to be occupied by a stopped car $1 - \langle v\rangle_n^j$ is divided by the empty space between the stopped cars and another one $T_w \times 1$, where 1 denotes the car's velocity; therefore, the density of the stopped cars follows the relation

$$\frac{N_{n0}^j}{L_n^j} = \frac{1-\langle v\rangle_n^j}{T_w \times 1}, \tag{13}$$

where N_{n0}^j is the number of stopped cars in the normal region, and $\langle v\rangle_n^j$ is the mean velocity of the cars in the high-density region of the normal area. Because of the flow limited by the speed limit zone, the average of velocity of the cars in the high-density region of the normal area reads

$$\langle v\rangle_n^j = \frac{1}{T_w + 1}. \tag{14}$$

Thus, the fraction of stopped cars in the high-density region of the normal area reads

$$\begin{aligned} n_0^j &= \frac{N_{n0}^j}{N_n^j} = \frac{\frac{1-\langle v\rangle_n^j}{T_w\times 1}L_n^j}{N_n^j} \\ &= \frac{1}{(T_w+1)\rho_n^j} = \frac{2(1-\sqrt{p})}{(2-\sqrt{p})^2}. \end{aligned} \tag{15}$$

According to the previous studies, the accident probability P_{ac}^{nj} in the high-density region of the normal area is proportional to the traffic flow and the stopped cars, and can be given as

$$P_{\mathrm{ac}}^{nj} = p'\langle J\rangle_s n_0^j = p'\frac{(1-\sqrt{p})^2}{(2-\sqrt{p})^2}. \tag{16}$$

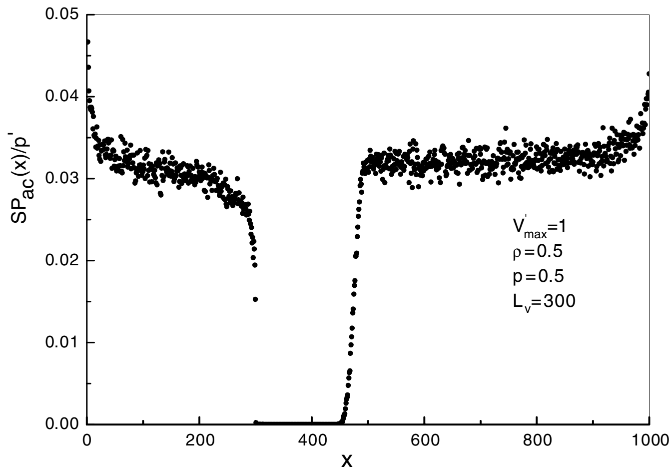

Figure 6. Distribution of the probability P_{ac} (scaled by p') in the nondeterministic NS model with the speed limit zone for the case of $p = 0.5$, $v_{\max} = 5$, $v'_{\max} = 1$ and $L_v = 300$.

Equation (16) demonstrates that $v_{\max}$ has no influences on the probability P_{ac}^{nj}. In the low-density region of the normal area, there is no stopped cars and no car accidents to occur, so the accident probability P_{ac}^{nf} in this region is equal to zero. Therefor, the probability P_{ac} for the occurrence of car accidents reads as

$$P_{\mathrm{ac}} = \frac{P_{\mathrm{ac}}^{v}\rho_v L_v + P_{\mathrm{ac}}^{nj}\rho_n^j L_n^j}{\rho L}. \tag{17}$$

Substituting equations (6), (10), (12), (16) into (17), we can obtain the relationship about P_{ac} to ρ. Comparison of our prediction for the probability P_{ac} with computer simulations shows excellent agreement in the density interval $\rho_{c1} \leqslant \rho \leqslant \rho_{c2}$ (see figure 5).

In the case of $\rho > \rho_{c2}$, the jams emerge on the whole road, the probability P_{ac} begins to decrease with further increase of ρ, as shown in figures 4 and 5. And the influences of the speed limit area on the probability P_{ac} cannot be observed when the density $\rho \to 1$. Therefore the accident probability P_{ac} is proportional to $\rho(1-\rho)$ when the density is near 1.

3.3. Position distribution of the probability P_{ac}

Because the speed limit zone results in the inhomogeneous density distribution of cars, unlike that of a model without a speed limit zone, the occurrence of car accidents depends on the sites. Figure 6 shows the relations of the accidents probability to sites in the case of $v_{\max} = 5$ and $v'_{\max} = 1$. The car density is in the interval $\rho_{c1} \leqslant \rho \leqslant \rho_{c2}$, which corresponds to the maximum flux of the speed limit zone and phase segregation into high density and low density in the normal region. As shown in figure 6, car accidents occur in both the speed limit zone and the normal region. In the high-density region car accidents occur, but in the low-density region no car accidents occur because of free flow and no stopped cars. The accidents probability $SP_{\mathrm{ac}}(x)$, which denotes the probability for car accident to occur per site per time step, exhibits very small fluctuation around a fixed value, except at the high-density boundary between the normal region and the speed limit zone. At the boundary, more cars are stopped due to the flow limit, hence cause the increase of the probability $SP_{\mathrm{ac}}(x)$. In the speed

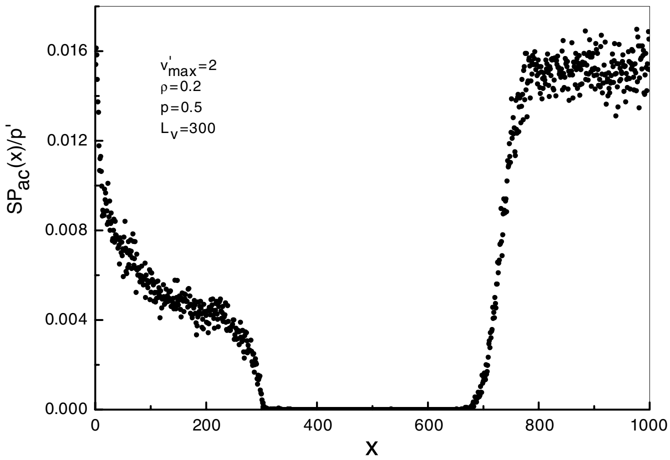

Figure 7. Distribution of the probability P_{ac} (scaled by p') in the nondeterministic NS model with the speed limit zone for the case of $p = 0.5$, $v_{max} = 5$, $v'_{max} = 2$ and $L_v = 300$.

limit zone and the high-density region within the normal region, the probability indicates that the same correlation of the spacetime exists.

However, for the case of $v'_{max} = 2$, the probability $SP_{ac}(x)$ exhibits different feature. As shown in figure 7, the probability $SP_{ac}(x)$ for car accident to occur per site per time step in the high-density region is larger than that in the speed limit region. Because of the conservation of the traffic flow in the entire system, the fraction of stopped cars in the high-density region is different from that in the speed limit zone. Therefore these results indicate that different correlations of spacetime exist between the speed limit region and the high-density region.

4. Summary

In this paper, we investigate the probability for the occurrence of car accidents in the NS model with a speed limit zone. In the real traffic systems, speed limit signs are usually posted to warn drivers of potential dangerous situations, such as sharp curves, tunnels, bridges, etc, requiring a reduction in the maximum speed. Because the speed limit zone results in distinct variety of traffic flow such as saturated flow observed a plateau region in the fundamental diagram and vehicle queuing, according to the previous studies of car accidents, the accidents probability is related to the stopped cars and traffic flow [14], the studies on the probability of the occurrence of car accidents can lead us to understand the traffic flow.

Numerical results show that the accidents probability P_{ac} in the model with the speed limit zone is equal to that in the NS model with the speed limit v'_{max} corresponding to the maximum speed of the speed limit zone in the deterministic cases. The probability P_{ac} is independent of the length of the speed limit zone. But in the nondeterministic cases, the probability P_{ac} is determined not only by the maximum speed but also by the length of the speed limit zone. In the nondeterministic NS model, the probability P_{ac} for car accidents to occur increases with the increase of the maximum speed of the speed limit zone in the low-density region. In the high-density region, the data for the probability P_{ac} shrink into one curve. And the probability for the occurrence of car accidents P_{ac} decreases with the increase of the length of the speed limit zone. The exception is the case of $v'_{max} = 1$. In the case of $v'_{max} = 1$, with the increase of the length L_v, the probability P_{ac} is suppressed in the low-density region, but enhanced in the interval between the high density and low density. The differences of the accident probability between the case of $v'_{max} = 1$ and $v'_{max} \neq 1$ are caused by the conditions of the occurrence of car accidents and the update ruled of the model.

The speed limit zone also leads to the inhomogeneous distribution of car accidents over the whole road. In the case of $v'_{max} = 1$, the probability for car accidents to occur per site per time step $SP_{ac}(x)$ exhibits very small fluctuation around a fixed value in the speed limit and the high-density region of the normal area. But in the case of $v'_{max} \neq 1$, i.e. $v'_{max} = 2$, the probability $SP_{ac}(x)$ for car accident to occur per site per time step in the high-density region is larger than that in the speed limit region.

A phenomenological mean-field theory is presented to describe the probability P_{ac} in the nondeterministic NS model. Theoretical analyses give an agreement with numerical results in the nondeterministic NS model with $v'_{max} = 1$. But in the nondeterministic NS model with $v'_{max} > 1$, explicit expressions about the probability P_{ac} which deserve further investigate cannot be obtained because of effects of long length of timespace correlations.

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